

# Chern-Simons Inflation and Baryogenesis

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(Dated: November 17, 2011)

We propose a model of inflation where the Chern-Simons interaction and vector fields play a central role in generating an inflationary epoch. As a result, in accord with the APS mechanism [5], the Sakharov conditions for baryogenesis are self-consistently satisfied, and we calculate the net baryon asymmetry index in terms of the time-dependent gauge configuration necessary for inflation, based on the chiral anomaly. Inflation begins with a large plasma density of interacting gauge fields and fermions, which interact through gravity and the Chern Simons term. The Chern-Simons term drives power from an initial white-noise spectrum of gauge fields into a narrow-band of superhorizon wave vectors. At the same time, the fermionic current and metric coupling amplifies the gauge field on superhorizon scales. This amplification of horizon sized gauge field produces the correct conditions to maintain more than 60 e-folds of inflation. Eventually the gauge field dissipates by producing the observed baryon asymmetry  $\frac{n_b}{s} \sim 10^{-10}$ , through the chiral anomaly and inflation ends.

## I. INTRODUCTION

The most successful paradigm of the early universe is cosmic inflation. Despite its utility in model building, a number of conceptual and technical challenges associated with inflation driven by fundamental scalar fields have been discussed in the literature [1]. Inflationary models driven from vector fields have been considered in the past beginning with the work of Ford [2] followed by other authors [3, 4]. Building on these past investigations, we present a model of inflation with new ingredients involving the interaction of CP-asymmetric abelian gauge fields and fermionic currents, as opposed to solely gauge fields, to generate a realistic inflationary epoch.

Moreover, this mechanism naturally satisfies the Sakharov conditions for baryogenesis during inflation by dynamically intraconverting the initial configuration of the CP-asymmetric gauge fields into the baryon asymmetry of the universe, building previous work of [5]. To be concrete, we explore a model of inflation that is generated from multiple copies of Abelian gauge fields interacting with fermions in an FLRW background. The model differs from previous attempts of gauge field inflation since it does not rely on any explicit gauge mass term nor non-trivial self interactions of the gauge fields to drive inflation.

We now wish to spell out the basic idea of the mechanism before its mathematical instantiation. In this model of inflation we find that the negative pressure equation of state responsible for inflation is generated by a nearly constant interaction energy between the gauge field and fermion current  $V_{int} \sim A_\mu \bar{\psi} \gamma^\mu \psi$  (as opposed to potentials that are purely functionals of gauge fields  $V(A_\mu A^\mu)$ ). We assume that the universe comprises of initial configuration of abelian gauge fields  $A_\mu^0$  and a coherent state of fermion currents  $J_\mu^0$ . In an expanding space-time, gauge field and fermion currents typically dilute. However, we find new solutions of the coupled field equations

demonstrating that the gauge fields are amplified due to a fermionic and metric source term. Eventhough the fermion current dilutes with cosmic expansion, the gauge field amplification will compensate and a constant interaction energy between the fermion and gauge field continues to persist within the horizon scale, which drives inflation.

It is well known that vector models of inflation generically lead to large anisotropies due to the spatial orientation of gauge fields. However, anisotropies can be naturally suppressed by employing  $N$  copies of gauge fields that are randomly distributed on the initial hypersurface at the beginning of inflation. As a result the anisotropies are suppressed by a factor of  $\frac{1}{\sqrt{N}}$ .

A crucial role that the Chern-Simons term plays in this model concerns one-loop quantum effects between gauge fields and fermions in the standard model (*i.e.* the Chiral Anomaly). Consequently, the Chern-Simons interaction is active during the inflationary epoch so as to naturally satisfy all three Sakharov conditions and can naturally end inflation by the production of an observed baryon asymmetry when the gauge field is converted into leptons through the Chiral-Anomaly[5]. Assuming an instantaneous reheating, we calculate the net baryon number to be  $\frac{n_b}{s} \sim 10^{-10}$  in terms of the gauge fields that sourced inflation, making a further connection between gauge field driven inflation and baryogenesis; a relation between the observed baryon asymmetry and the initial density of gauge fields necessary for the onset to inflation.

## II. THE THEORY

In this model, we assume that the early universe is dominated by  $N$  copies of an abelian gauge field interacting with a fermionic current. The gauge field  $A_\mu^A$  has a  $U(1)^N$  symmetry with an index,  $A=1..N$ . Generic string compactifications can easily accomodate multiple copies

of gauge fields. In particular, one can obtain embeddings by stacking  $N$  coincident D-branes which leads to a  $SU(N)$  gauge theory. Symmetry breaking patterns exist where the gauge theory breaks to an abelian subgroup  $SU(N) \rightarrow U(1)^{N-1}$ ; we refer the reader to the review by Langacker and references within for more details [6]. These extra  $U(1)$ 's are anomalous and the Chern-Simons term is necessary for gauge anomaly cancellation due to the Green-Schwarz mechanism [7] [32]. The effective field theory that we consider for inflation is [33]:

$$S = S_D + \int_{\mathcal{M}_4} d^4x \sqrt{-g} \left[ \frac{M_p^2 R}{8\pi} - \frac{1}{2} \partial_\mu \theta \partial^\mu \theta + m^2 \theta^2 + \right. \\ \left. - \frac{1}{4} \text{Tr}[F_{\alpha\beta} F^{\alpha\beta}] + \frac{\theta}{4M_*} \text{Tr}[F_{\alpha\beta} \tilde{F}^{\alpha\beta}] + q \text{Tr}[A_\mu \mathcal{J}_5^\mu] \right], \quad (1)$$

where  $S_D$  is the covariant Dirac action:

$$S_D = \int d^4x \sqrt{-g} [-i \bar{\psi} \not{D} \psi + \frac{3}{M_p^2} (\bar{\psi} \gamma^I \gamma_5 \psi)^2 + q \bar{\psi} \gamma^I e_I^\mu \gamma_5 \psi A_\mu]$$

We will consider the dynamics of the fermions for self-consistency in Appendix A.  $\mathcal{M}_4$  is a 4D space-time manifold whose signature is  $\text{diag}(-1, 1, 1, 1)$ . The tensor  $F_{\mu\nu}^A = \partial_{[\mu} A_{\nu]}^A$  is the field strength tensor of  $A_\mu^A$ . We denote the sum over the indices labeling the  $U(1)$  copies as  $\text{Tr}[\dots]$ , while  $q$  stands for a dimensionless coupling constant. The axial current for spinors is  $\mathcal{J}_5^\mu \equiv \bar{\psi} \gamma^\mu e_I^\mu \gamma_5 \psi$ . Here  $\psi$  and  $\bar{\psi}$  are Dirac spinors,  $\gamma^I$  with  $I = 0, \dots, 3$ , and  $\gamma_5$  are Dirac matrices. Finally we refer to  $M_p$  as the Planck energy,  $M_*$  is the mass scale identified with the UV cutoff scale of the effective field theory, and  $\theta$  is responsible for  $CP$  violation.

For the purpose of efficiency of the presentation we will evaluate the dynamics of  $\theta$  field in Appendix A and show that its energy density is twenty orders of magnitude smaller than the gauge-fermion interaction, making it insignificant for driving inflation. In this model, it is the Chern-Simons and fermion current interaction with the gauge field that conspires to yield the negative pressure equation of state in order to initiate and drive inflation. To establish this, we will first compute the energy momentum tensor of the Lagrangian of eq(1):

$$\tilde{\mathcal{L}} = \text{Tr} \left[ -\frac{1}{4} g_{\alpha\gamma} g_{\beta\delta} F^{\gamma\delta} F^{\alpha\beta} + \frac{1}{4} \frac{\theta}{M_*} F_{\alpha\beta} \tilde{F}^{\alpha\beta} + q A_\mu \mathcal{J}_5^\mu \right].$$

Using the relation  $T^{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta \sqrt{-g} \tilde{\mathcal{L}}}{\delta g_{\mu\nu}}$ , we find that the energy-momentum tensor is:

$$T_\nu^\mu = -\delta_\nu^\mu \tilde{\mathcal{L}} + \text{Tr} \left[ F_{\alpha\nu} F_{\rho\beta} g^{\alpha\beta} g^{\rho\mu} - \frac{\theta}{4M_*} F_{\alpha\beta} \tilde{F}^{\alpha\beta} \delta_\nu^\mu - A_\nu \mathcal{J}_5^\mu \right] = \\ = \text{Tr} \left[ -\delta_\nu^\mu A_\rho \mathcal{J}_5^\rho + F_\alpha^\mu F_\nu^\alpha - \frac{1}{4} \delta_\nu^\mu g^{\alpha\rho} g^{\beta\sigma} F_{\alpha\beta} F_{\rho\sigma} - A_\nu \mathcal{J}_5^\mu \right]. \quad (2)$$

We will now demonstrate that the purely isotropic part of  $A \cdot \mathcal{J} \equiv \text{Tr}[A_\mu \mathcal{J}_5^\mu]$  in the energy-momentum tensor dominates over the anisotropic terms for large values of  $N$ . The other anisotropic terms in the energy-momentum

tensor will be suppressed for the following reasons. First, the anisotropy due to the  $F_{\alpha\mu} F_\nu^\alpha$  term will be negligible because the corresponding electric and magnetic fields redshifts as  $a^{-4}$  during inflation according to the RHS of eq. (13). Also, the anisotropy due to  $\text{Tr}[A_\nu \mathcal{J}_5^\nu]$  term can be suppressed by two mechanisms. First, if there are  $N$  copies of randomly oriented gauge fields and currents, then the correction to the energy momentum tensor will be [3, 4]:

$$T_{ij} = \sum_{A=1}^N A_{(i}^A \mathcal{J}_{j)}^A \simeq \frac{N}{3} A \cdot \mathcal{J} \delta_j^i + O(1) \sqrt{N} A_{(i} \mathcal{J}_{j)}. \quad (3)$$

The second term arises from the stochastic random distribution in directions of the fields that are non-vanishing if  $i \neq j$ . Notice that the ratio between the anisotropic and isotropic part of the averaged stress tensor scales as (3)  $\frac{|T_A|}{|T_I|} \sim \frac{\sqrt{N} A \cdot \mathcal{J}}{N A \cdot \mathcal{J}} \sim \frac{1}{\sqrt{N}}$ . For values of  $N \sim 10^5$  we can treat the anisotropic part of the energy-momentum tensor as a perturbation.

Finally, another isotropizing mechanism is to have a triad of gauge fields that are mutually orthogonal  $A_\mu^A = \delta_\mu^A A(\eta, x)$  [4]. For more details about orthogonal gauge fields we refer the reader to [8]. Note that we can incorporate the orthogonality condition in our model if  $N = 3$ . However, this particular value of  $N$  introduces a fine-tuning problem. Whereas  $SU(N)$  gauge theories in the large  $N$  limit are quite natural in string theory [9].

We now focus our attention on the isotropic part of the energy momentum tensor(2) which is  $\varrho = (\vec{E}^2 + \vec{B}^2)/(2a^4) + A \cdot \mathcal{J}$ . Where we have introduced the definitions of electric field  $\vec{E} \equiv \vec{A}$  and magnetic field  $\vec{B} \equiv \vec{\nabla} \times \vec{A}$ . It is now clear that the energy density has two components. The first component is the electromagnetic part that redshifts as  $a^{-4}$ , and the second term involves a gauge-fermion-current interaction that we are going to show to be roughly constant and responsible for inflation. We shall now demonstrate that the coupled field equations indeed yield an inflationary epoch if certain initial conditions are satisfied.

### III. THE INFLATIONARY EPOCH AND INITIAL CONDITIONS

In what follows, we derive solutions to the field equations for the gauge field coupled to both the metric and the fermionic current. The key to generating inflation is that the scale factor exhibits inflationary behavior when the gauge field and the fermionic current interaction remain nearly constant during inflation due to backreaction of the gauge field.

First, we seek to find solutions of the gauge field in the FLRW background by varying the action with respect to  $A_\mu$

$$\delta \gamma^{[\beta} \partial^{\alpha]} F_{\alpha\beta} - \varepsilon^{\alpha\beta\mu\nu} \partial_{[\alpha} \theta \delta_{\beta]}^\gamma F_{\mu\nu} / (4M_*) + \sqrt{-g} \mathcal{J}_5^\gamma = 0. \quad (4)$$

In the Coloumb gauge  $A_\mu = (0, A_i)$ , equation (4) yields

$$\vec{A} - \nabla^2 \vec{A} + \vec{\nabla} \times \vec{A} \dot{\theta}/M_* - a^4 \vec{\mathcal{J}}_5 = 0, \quad (5)$$

We seek to find self consistent solutions by working in conformal coordinates and assuming the expansion of the Universe to be given by a quasi-de Sitter phase, namely  $a(\eta) = a_0/[1 - H(\eta - \eta_0)]$  and  $\eta_0$  the initial time. In (5)  $\dot{\phantom{x}} = d/d\eta$ , while  $\nabla_i = \partial/\partial x^i$  and  $\mathcal{J}_5^i$  stands for the spatial part of the fermionic axial current. Without loss of generality, using circular polarization vector fields and setting  $A_3$  to be vanishing, we can write the field equations in terms of Fourier modes  $\vec{A}(x, \eta) = \int d^3k \sum_h A(\eta, k)_h \epsilon_h(k) e^{ikx}$ , where  $h = \pm 1$  denotes the two possible helicities. Our field equation for the gauge field then simplifies to

$$\ddot{A}(\eta, k)_h + k^2 A(\eta, k)_h = -h k A(\eta, k)_h \dot{\theta}/M_* + a^4 \mathcal{J}_h, \quad (6)$$

where the spatial fermionic current is given by

$$\sqrt{2} \mathcal{J}_h = J_h/a = (\bar{\psi}(\gamma_1 + hi\gamma_2)\gamma_5\psi)/a. \quad (7)$$

Here, the spatial current  $J_h = \bar{\psi}(\gamma_1 + hi\gamma_2)\psi$  is shown to be a constant in Appendix A. We can then rewrite (6) as

$$\ddot{A}(\eta, k)_h + k^2 A(\eta, k)_h = -h k A(\eta, k)_h \dot{\theta}/M_* + a^3 J_h. \quad (8)$$

The general solution for the left-handed gauge field is then [34]:

$$A(\eta, k)_- = A_-^0 \cosh(\beta_k \eta) + \tilde{A}_-^0 \sinh(\beta_k \eta) + \Xi[H, \beta, \tilde{J}_-, \eta], \quad (9)$$

where  $A_-^0$  and  $\tilde{A}_-^0$  are determined by the initial conditions,  $\tilde{J}_\pm = a_0^3 J_\pm$  and the growth factor is  $\beta_k^2 = k(\dot{\theta}/M_* - k)$ . We find that the particular solution is:

$$\Xi[H, \beta, \tilde{J}_-, \eta] = \frac{\tilde{J}_-}{2H^2(1-H\eta)} + \frac{\beta \tilde{J}_-}{4H^3} \Psi(x) \Big|_{x=-\frac{\beta}{H}(1-H\eta)}^{x=\frac{\beta}{H}(1-H\eta)},$$

with  $\Psi(x) = e^x \int_{\infty}^x \frac{e^{-t} dt}{t}. \quad (10)$

We will have exponentially growing fields provided that  $\beta_k$  is real (or  $k < \dot{\theta}/M_*$ ) [35]. The important behavior for the gauge field is in the particular solution which gets amplified as  $a(\eta)$  due to the factor  $\tilde{J}_-/[2H^2(1-H\eta)]$ .

The gauge field,  $A(\eta, k)_+$ , of opposite helicity also grows as  $a(\eta)$  *i.e.*

$$A(\eta, k)_+ = A_+^0 \cos(\gamma_k \eta) + \tilde{A}_+^0 \sin(\gamma_k \eta) + \Sigma[H, \gamma, \tilde{J}_+, \eta], \quad (11)$$

in which  $\gamma_k^2 = k(\dot{\theta}/M_* + k)$ , and the particular solution  $\Sigma[H, \gamma, \tilde{J}_+, \eta] \equiv \Sigma$  is

$$\Sigma = \frac{\gamma \tilde{J}_+}{2H^3} \left( \frac{1}{y} + \sin(y) \int_{-y}^{\infty} \frac{\cos t dt}{t} + \cos(y) \int_0^y \frac{\sin t dt}{t} \right), \quad (12)$$

with  $y = (1 - H\eta)\gamma/H$ . Notice that the particular solution (12) grows as  $a(\eta)$  due to the factor  $\tilde{J}_+^0/[2H^2(1 - H\eta)]$  for all values of  $k$ . Moreover, (12) also exhibits a peak around a resonant momentum  $\tilde{k} = \frac{\dot{\theta}}{2M_*}$ , which arises from the Chern-Simon's interaction.

Now that we have the Fourier modes of the gauge field,  $A(\eta, k)_\pm$  we can integrate of all momenta to obtain its spatial dependence:  $\vec{A}(x, \eta) = \int d^3k A(\eta, k)_\pm e^{ikx}$ . In the next section we will perform this integral explicitly in order to show that the term  $A \cdot \mathcal{J}$  is nearly constant.

#### IV. CONSISTENCY OF INFLATIONARY DYNAMICS

We now turn our attention to the Einstein Equations and seek a consistent inflationary solution [36]. The  $G_{00}$  component of the Einstein Equations gives the first Friedmann equation:

$$3 \frac{\dot{a}^2}{a^4} = \frac{8\pi G}{a^4} (E_+ E_- + B_+ B_-) + 8\pi G (A_+ \mathcal{J}_- + A_- \mathcal{J}_+). \quad (13)$$

The coupled system can be solved if the interaction term  $A \cdot \mathcal{J}$  is nearly spatio-temporally constant during inflation. The fact that  $\mathcal{J}_+$  and  $\mathcal{J}_-$  dilutes as  $1/a$  might raise concern, but notice that for both the gauge field components, the dominant term grows proportional to  $a(\eta)$ , as is evident from (9) and (11) [37] and dominates the r.h.s of (13). If  $A \cdot \mathcal{J}$  is constant we can solve the Friedmann equation (13) to find an inflationary scale factor,

$$a(\eta) = a_0/[1 - H(\eta - \eta_0)], \quad (14)$$

where  $a_0$  is a normalization factor such that  $a(\eta_0) = a_0$ , and we have defined the Hubble parameter to be  $M_p H \simeq \sqrt{(A \cdot \mathcal{J})_{\eta_0}} a_0$ . We can easily obtain the comoving scale factor  $a(t)$  using the map  $\partial\eta = \partial t/a$ , which is  $a(t) = a_0 \exp Ht$ , which gives rise to exponential growth in the scale factor. With the choice of initial conditions such that  $a(\eta)/a_0 = (1 - H\eta)^{-1} = \exp Ht$ , the time coordinate is conformally mapped to the bounded range  $\eta \in [0, H^{-1})$ .

Returning to the Friedmann equation (13) we can check the validity of our *ansatz* for spatial constancy of  $A \cdot \mathcal{J}$  by using the solutions of the gauge fields (9, 11) [38]. Taking the Fourier transform of  $A(\eta, k)_\pm$  yields the energy density of  $A \cdot \mathcal{J}$ :

$$\int_{-\mathcal{K}}^{+\mathcal{K}} \frac{dk}{2\mathcal{K}} (1 - H\eta) \left\{ \tilde{J}_-^0 \left[ A_+^0 \cos(\gamma_k \eta) + \Sigma[H, \gamma_k, \tilde{J}_+, \eta] \right] + \tilde{J}_+^0 \left[ A_-^0 \cosh(\beta_k \eta) + \Xi[H, \beta_k, \tilde{J}_-, \eta] \right] \right\} e^{ikz} = A \cdot \mathcal{J}, \quad (15)$$

where we have normalized the integration over  $k$  by the factor  $2\mathcal{K}$ . Since we are working with an effective field theory, we must perform momentum integrations up to a cut-off  $\mathcal{K} \simeq M_*$ . If we define the momentum scale

$\bar{k} = \dot{\theta}/M_*$ , we can express the Fourier transform in eq (15) in a compact way and recognize that  $\bar{k}$  contributes a spatial dependence of the form  $\exp[\pm i\bar{k}z]$ . Notice that the result of integrating over  $k$  yields the rapidly decreasing functions of time,  $f_i(\eta)$ . Therefore, the energy density (15) becomes:

$$A \cdot \mathcal{J} \sim 2 \frac{\tilde{J}_- \tilde{J}_+}{H^2} \frac{\sin(\mathcal{K}z)}{\mathcal{K}z} + \frac{1}{a(\eta)} \left\{ J_- A_+^0 f_1(\eta) + \tilde{J}_+ A_-^0 f_2(\eta) + \frac{\tilde{J}_- \tilde{J}_+}{H^2} f_3(\eta) \right\}. \quad (16)$$

The second term on the RHS of (16) are exponentially time decaying terms in the quasi de-Sitter background and can be disregarded, since they will decay after the onset of inflation. We immediately see that when  $\mathcal{K}z < 1$  the first term in (16) becomes a constant,  $2 \frac{\tilde{J}_- \tilde{J}_+}{H^2} \frac{\sin(\mathcal{K}z)}{\mathcal{K}z} \rightarrow 2 \frac{\tilde{J}_- \tilde{J}_+}{H^2}$ , and dominates the energy-momentum tensor. This condition corresponds to vector modes inside the Horizon,  $z \leq \frac{1}{\mathcal{K}}$ . In this regime the FLRW equations become:

$$M_p^2 H^2 \simeq 2 \frac{\tilde{J}_- \tilde{J}_+}{H^2}. \quad (17)$$

Recalling that  $\tilde{J}_\pm = a_0^3 J_\pm$ , the above equation consistently leads to inflation provided that  $\tilde{J}_-/H \simeq \tilde{J}_+/H \simeq 10^{-5} M_p^2$ ,  $A_-^0 \simeq A_+^0 < 10^{-5} M_p$  and  $a_0 = 10^5$ . Also, as we will analyze in the appendix, self consistency with the field equations requires  $\dot{\theta}/M_* = \bar{k} \simeq H \simeq M_*$ . With the above values for the initial gauge fields, we find values of the fermionic current to be  $J_+ \simeq J_- \simeq 10^{-25} M_p^3$ , namely  $\mathcal{J}_+(\eta_0) \simeq \mathcal{J}_-(\eta_0) \simeq 10^{-30} M_p^3$ . Furthermore, in Appendix B we analyze the gauge invariant vector perturbations induced by fluctuations of gauge field  $\delta A(\eta, k)$  and show that they decay as  $\frac{1}{a^2}$ .

We have demonstrated that the solution of the gauge field in a time dependent background self-consistently leads to an energy-density  $A \cdot \mathcal{J}$  that is constant in the Hubble radius at the beginning of inflation. The inflationary solution is self consistent for an initial density of fermion current and gauge field that are constant in a region inside the initial Hubble radius.

## V. INFLATIONARY LEPTOGENESIS

Primordial nucleosynthesis and the recent determination of the cosmological parameters from the cosmic microwave background observations by the WMAP satellite require an excess of baryon to entropy density ratio [13] in the universe to be

$$\frac{n_B}{s} = (6.5 \pm 0.4) \times 10^{-10}, \quad (18)$$

where  $n_B = n_b - n_{\bar{b}}$  and  $s$  is entropy density of radiation. In this section we argue that this inflationary mechanism

provides the possibility of generating the baryon asymmetry during the inflationary epoch. The baryogenesis mechanism, while compelling, will need additional ingredients, such as a rapid reheating mechanism before we can claim with more confidence that baryogenesis actually occurs. In what follows we present encouraging clues that the Sakharov conditions occur quite naturally during inflation. We will explicitly calculate the net lepton number produced by the gauge fields during inflation, using the one-loop ABJ anomaly.

Over forty years ago, Sakharov stated the three necessary conditions to generate a matter-antimatter asymmetry dynamically from symmetric initial conditions [14]:

1. The vertices in the model violates baryon/lepton number.
2. CP should be violated in the baryon/lepton number vertices.
3. CP and baryon number violating interactions should be active when the universe is out of thermal equilibrium.

However, the Standard Model of Particle Physics (SM) cannot account for the observed baryon asymmetry because baryon number violating interactions in the SM are loop suppressed [15, 16]. Also, the only source of CP violation in the hadronic sector is in the Dirac phase of the CKM mixing matrix, which is too small to generate the baryon asymmetry observed today [17]. Assuming that the scale of inflation is larger than TeV, the out-of-equilibrium condition for baryogenesis can be created at phase transitions or through decay of massive particles. The most attractive choice for a phase transition is that associated with electroweak symmetry breaking. However, that phase transition is probably not strongly first-order [15, 16].

Since the 1980's, it has been known that the weak interactions contain processes, mediated by *sphalerons* ( $SU(2)$  instantons), which interconvert baryons and leptons and are thermally activated at temperatures greater than 1 TeV. Thus, we can also create the baryon asymmetry by creating a net lepton number at high temperature through out-of-equilibrium and CP-asymmetric processes [17, 18]. These types of scenarios are known as *leptogenesis*. In what follows, we demonstrate that the Sakharov conditions are interrelated due to the CP violating gauge field that is responsible for sourcing inflation itself. We then calculate the net lepton asymmetry produced by the end of inflation.

### A. CP Violation

It was shown by [5] that inflation provides a natural arena for satisfying the three Sakharov conditions for baryogenesis [39]. In [5], the lepton asymmetry was generated by CP violating gravitational waves during inflation. In this

model, however, it is the gauge field rather than gravitational waves that is responsible for the lepton asymmetry. Specifically, the interaction between the pseudoscalar and Chern-Simons term in (5) sources CP asymmetric gauge field configurations. The pseudoscalar coupling creates left and right asymmetry in the circular polarized gauge fields,  $A_+, A_-$ . The gauge fields that are generated no longer have definite transformations under CP:

$$[\hat{CP}]\vec{A}_+ \neq \vec{A}_- \quad (19)$$

This condition can explicitly be checked by noticing that the left and right-handed gauge fields (9) and (11) have unequal amplitudes. A parity transformation exchanges the handedness of a gauge field plane wave but does not transform the amplitude. Note that this form of CP violation is not explicit but dynamical since the birefringent amplitudes are sourced by the coherent evolution of the pseudoscalar,  $\theta$ , during inflation, which arises from solutions of  $\theta$  presented in Appendix A.

### B. Lepton Number Violation

In this model, lepton number is violated at one-loop through the chiral triangle anomaly in the standard model [22]. Through the ABJ anomaly, the gauge field intraconverts itself into a net fermion number during the inflationary epoch via the following equation:

$$\nabla_\mu \mathcal{J}_l^\mu = F_{\alpha\beta} F_{\mu\nu} \epsilon^{\alpha\beta\mu\nu} / (32\pi^2), \quad (20)$$

where the leptonic current is:

$$\mathcal{J}_l^\mu = \bar{l}_i \gamma^I e_I^\mu \gamma_5 l_i + \bar{\nu}_i \gamma^I e_I^\mu \gamma_5 \nu_i, \quad (21)$$

and we have assumed that the leptons couple only to the hypercharge sector,  $U(1)_Y$  of the minimal standard model. Since we have extended the gauge sector to include  $N$  copies of gauge field, we are assuming that these abelian factors couple solely to leptonic charge in the minimal standard model. In general we have the freedom to couple the gauge fields to other states that are not charged under the weak-hypercharge assignment (such as a dark sector). This non-minimal charge assignment may connect our model to a mechanism of dark matter production at the end of inflation [40]. While this other possibility is to be pursued in an independent work, we wish to make a few remarks on non-standard charge assignments.

It is interesting that the many copies of  $U(1)$  necessary for our model to generate isotropy can be related to having extra  $Z'$  boson(s), if we make a non-minimal assignment of the fermions to extra  $U(1)$ 's then they can be associated as dark matter, which was considered by Brahm and Hall [10]. We can explicitly implement the coupling of leptons to one of  $U(1)_Y$  as a the weak hypercharge current in the standard model and the other  $U(1)^n$  copies as a set of  $Z'$  bosons. In this coupling,

the standard model neutral current interactions of the fermions will be in standard form:

$$\mathcal{L}_{NC}^{SM} = g \mathcal{J}^\mu_{em} A_\mu + g_1 \mathcal{J}^\mu_1 Z_{1\mu}^0 \quad (22)$$

where  $g$  and  $g_1$  are the  $SU(2)$  and  $U(1)_Y$  gauge couplings;  $A_\mu$  and  $Z_{1\mu}^0$  are the photon and massive  $Z$ -boson. On the other hand, the extra  $U(1)$ 's can couple to the lepton current via the identification with extra  $Z'$  bosons. Following Langacker and references therein [6], we can modify the leptonic couplings to the the extended interactions of an  $SU(2) \times U(1)_Y \times U(1)'^n$  (with  $n \geq 1$ ) gauge sector:

$$\mathcal{L}_{NC} = g \mathcal{J}^\mu_{em} A_\mu + \sum_{\alpha=1}^{n+1} g_\alpha \mathcal{J}^\mu_\alpha Z_{\alpha\mu}^0, \quad (23)$$

where  $g_1, Z_{1\mu}^0$  and  $\mathcal{J}^\mu_1$  are respectively the gauge coupling, boson and coupling of the standard model. While,  $g_\alpha, Z_{\alpha\mu}^0$  are the gauge couplings and bosons for the additional  $U(1)$ 's. In this non-standard coupling we will have to consider the production of leptons via the decay of the  $Z'$  bosons, which is an issue that requires a detailed analysis on its own right.

We expect the non-standard couplings of the extra gauge fields to alter our mechanism. We speculate that if the couplings are the same order as that of the standard model, then dark fermions will also be produced due to the triangle loop anomaly in the form of eq. (20). It is then plausible that the non-standard couplings to a dark sector could provide a potential mechanism for dark matter production by the end of inflation via a kinetic mixing between the standard model  $U(1)_Y$  and the  $U(1)'$  sectors. Such scenarios called *Secluded WIMP Dark Matter* have been discussed by Pospelov et. al. [11], and it will be interesting to connect those models to our scenario in future work; although this is a topic outside the scope of our present work [41] [42].

We now show that net lepton number resulting from eq (20) is non-vanishing precisely because the gauge field configuration that is responsible for inflation is also CP violating. This relationship between two Sakharov conditions is unique to our model. We can immediately calculate the amount of matter asymmetry produced from the gauge fields that initiated inflation through eq. (20). Through the triangle interaction the gauge-field converts itself into a net lepton number accumulated throughout the inflationary epoch, and (20) becomes:

$$n = \int_0^{H^{-1}} \frac{d\eta}{a^3(\eta)} k (\dot{A}(\eta, k)_+ A(\eta, k)_+ - \dot{A}(\eta, k)_- A(\eta, k)_-). \quad (24)$$

From (9) and (11), using the time dependent solutions of the gauge field  $A(\eta, k)_\pm$  and of currents necessary for inflation, we find  $n \sim 10^{-15} M_p^3$ .

We can use the Friedmann equation to find the entropy density,  $s = 1.8 g_* n_\gamma$ , where  $g_*$  is the effective number of massless degrees of freedom (e.g. 100 for the MSSM) and

$n_\gamma = 1.28 g_*^{-3/4} (H M_p)^{3/2}$ . Hence, we calculate a realistic value for the baryon asymmetry index  $n/s \sim 10^{-10}$ . Note that the time evolution of the gauge field,  $\dot{A}_\pm$  is crucial to obtain the correct lepton-number, which shows the interesting relation between the dynamics of the gauge field during inflation and leptogenesis. We intend to pursue the real time dynamics of the lepton number production in a forthcoming paper.

### C. Out of Equilibrium

Lepton number and CP violation occur simultaneously during inflation through the gauge-fermion vertex and triangle loop diagram respectively. However, the exponentially expanding background space-time renders the universe to be far out of equilibrium with the fermion production. Furthermore, because we are generating the Sakharov conditions during inflation, this leptogenesis mechanism depends on the details of a reheating mechanism to conserve the net-leptons produced during inflation. We assumed a spontaneous reheating [25], however if reheating occurs slowly to a reheating temperature  $T_r < T$ , then the baryon asymmetry index,  $\frac{n}{s}$  will be diluted by a factor  $T_r/T$ . Therefore, to firmly establish that we have a successful baryogenesis mechanism will require a detailed analysis of reheating in this model; a topic that we leave for future work.

## VI. END OF INFLATION

In most scalar models of inflation, inflation ends when the inflaton no longer satisfies the slow-roll conditions. Our model is similar in that the end of inflation is triggered by the late time oscillation of the  $\theta$  field about its minimum. Recall, that inflation is driven by a nearly constant interaction energy between the gauge-fields and fermionic currents, which dominates the energy momentum tensor. The solution of the field equations for the gauge fields responsible for inflation (eq [8]) arose from a phase where the velocity of the  $\theta$  field is slowly-rolling (*i.e.*  $\dot{\theta} \sim \text{const}$ ):

$$\ddot{A}(\eta, k)_h + k^2 A(\eta, k)_h = -h k A(\eta, k)_h \dot{\theta} / M_* + a^3 J_h. \quad (25)$$

Eventually, the  $\theta$  field, which couples to the vector field, evolves to the bottom of its potential, begins to oscillate, and the slow-roll condition is violated,  $\epsilon = \frac{M_p^2}{2} \left( \frac{V'(\theta)}{V(\theta)} \right)^2 \geq 1$ .

Eventhough the scalar potential does not source inflation, its eventual oscillatory coupling to the gauge field will end inflation. Hence, following the work of Kofman et al. and Traschen et. al [26, 27], at late times the scalar field approaches sinusoidal behavior:

$$\theta = \frac{M_p}{3\pi m t} \sin(mt) \quad (26)$$

resulting in a Mathieu equation which yields a resonant particle production:

$$\ddot{A}(\eta, k)_h + k^2 A(\eta, k)_h + \Phi_0 \sin(mt) A(\eta, k)_h + a^3 J_h = 0. \quad (27)$$

where the amplitude is  $\Phi_0 = \frac{k M_p}{3\pi M_* t}$ . Notice that we can use (26) because of the fast roll condition ( $\epsilon > 1$ ), as the kinetic term  $\dot{\theta}$  dominates over Chern-Simons potential.

As a result of the oscillation of the  $\theta$  field about its minimum, the gauge field ceases to have an exponential time dependent growth to sustain inflation. This happens because the Chern-Simons coupling of  $\theta$  to the gauge field mimics a Mathieu equation (27) for the gauge field. This leads to resonant particle production similar to what happens in parametric resonance in preheating. However according eqs. (27) and (24), CP asymmetric parametric resonance and the gauge field will decay into fermion number via the Chiral anomaly. As a result, the oscillation of the  $\theta$  field triggers a sequence of decay of gauge field into fermions that ends inflation. Currently one of us and fellow collaborators are pursuing a preheating mechanism based on the coupled dynamics of the fermion, gauge field and oscillating  $\theta$  field numerically [28].

## VII. CONCLUSION AND DISCUSSION.

In this work we demonstrate that it is possible to obtain an epoch of cosmic inflation from abelian gauge fields sourced by the Chern-Simons current. This interaction leads to an exponential time dependence of the gauge field that compensates the diluting fermionic current to give a nearly constant energy density  $\rho_{AJ} \sim A \cdot \mathcal{J}$ . Power is dumped into a narrow band of modes, which drives inflation. Furthermore, we demonstrate that the CP violating gauge fields responsible for inflation can get interconverted into a net lepton number at the end of inflation. We calculate that this number gives a realistic lepton asymmetry. However, we assumed that the entropy production occurs from spontaneous reheating. We did not pursue a reheating mechanism inherent to this model. We have argued that parametric particle production of the gauge particles due to coherent oscillations of the  $\theta$  field triggers the end of inflation, which could yield a preheating phase, and we leave this numerical analysis for future work.

We plan to pursue the genericity of the assumptions of our initial conditions by implementing our mechanism in a cosmological fermionic bounce mechanism, which may set up the initial condition for the gauge and fermion field. Some of us are currently pursuing this question of generating an initial fermion current in the context of a bouncing cosmology where a fermionic current could bounce from a contracting FLRW to an expanding FLRW, by using techniques in Loop Quantum Cosmology developed in [29].

Finally, an explicit analysis of the generation of scale

invariant spectrum of density perturbation is initiated and we have found that both tensor and vector metric perturbations do not diverge. However a more detailed perturbation analysis is necessary and is complicated by the fact that in the presence of spatial components of a vector field, the decomposition theorem is violated. In particular, metric vector perturbations show up in the scalar perturbation equations [8]. However, this coupling may lead to observable non-Gaussianity effects in the CMB, and we leave this analysis for a future work [23]. In closing, we believe that this model of inflation may play a role in connecting cosmological observables to particle physics in new ways, such as the baryon asymmetry index with the measure of anisotropy and non-Gaussianity in the CMB.

### Appendix A: Consistency of the Analysis

We address now the evolution and issue of backreaction on the  $\theta$  field due to the Chern-Simons term. Eventually, the Chern-Simons term could provide a back-reaction potential for  $\theta$  that would be responsible for the non-constancy of  $\dot{\theta}/M_*$ . However, if we use the values of the vector field necessary for 60-e-folds of inflation we obtain that the maximum value of the backreaction for the Chern-Simons term is at the beginning of inflation. And in fact  $F_{\alpha\beta}\tilde{F}^{\alpha\beta}|_{\eta_0} = \vec{E} \cdot \vec{B}|_{\eta_0}/a(\eta_0)^4 \sim 10^{-20} M_p^4$  is suppressed by  $e^{-240}$  at  $\eta_f$ . Thus the Chern-Simons term would provide a negligible back-reaction to  $\theta$ . It is straightforward to check this statement by working in comoving coordinates and then pull-back the result in conformal coordinates using  $d\theta/d\eta = a(t)d\theta/dt$ . From (1) the time evolution of  $\theta$  in a homogenous FLRW metric with  $a=a_0 e^{Ht}$  is determined by

$$\ddot{\theta} + 3H\dot{\theta} + 2m^2\theta = \frac{\vec{E} \cdot \vec{B}}{4a^3 M_*}. \quad (\text{A1})$$

We study the general solution of (A1) ignoring the Chern-Simons term. The conformal coupling with the metric provides a Hubble friction term  $3H\dot{\theta}$  responsible for a dramatic decay of  $\dot{\theta}/M_*$  in both conformal and comoving time. However, the potential acts to prevent the decay in conformal time of  $\theta$  during inflation, provided that  $m = H$ , namely  $m$  is order GUT scale. With an appropriate choice of the initial condition, the general solution comoving coordinates is  $\theta = -\theta_0 e^{-Ht}$ . In conformal coordinates, such a solution would give the constant value  $d\theta/d\eta = \theta_0 H$ . As we have set up  $H \simeq M_*$ , the requirement in conformal coordinates  $\dot{\theta}/M_* \simeq 10^{-5} M_p$  is satisfied provided that  $\theta_0 \simeq 10^{-5} M_p$ . Now we solve for  $\theta$  with the Chern-Simons term present, which slowly varies during inflation. Therefore, the particular solution to (A1) becomes

$$\theta_p = \frac{(\vec{E} \cdot \vec{B})_0}{2a_0^3 H^2 M_*} e^{-3Ht}, \quad (\text{A2})$$

which at initial time of inflation contributes to the value of  $\dot{\theta}/M_*$  with a term  $\dot{\theta}_p/M_* = 10^{-20} M_p$ , after washed out by a factor  $a^2(\eta)$ . And the contribution to the minimum of the field from  $\theta_p$  will be  $\theta_p(0)/M_* = 10^{-15}$ , thus negligible within respect to the general solution to (A1). Therefore at initial time we find the ratio

$$\frac{V(\theta_0)}{(A \cdot \mathcal{J})_0} = \frac{m^2 \theta_0^2}{(A \cdot \mathcal{J})_0} \simeq 10^{-10} \quad (\text{A3})$$

that is exponentially suppressed at later times. It follows that potential scalar field  $\theta$  does not directly contribute to driving inflation.

Finally, we consider an analysis of the fermionic dynamics in (1). In our model, we argue that due to quantum squeezing, our fermions exist as a coherent state. Therefore, we must evaluate VEV of current operator in the Coherent state, which we will discover leads to the  $\frac{1}{a}$  redshift of the fermion current. The use of coherent states rather than free-states is crucial in our mechanism. Free-states can be relevant when evaluating transition amplitudes between free-asymptotic particle states. In turn, coherent states are applied to evaluate the expectation value of quantum fields, *e.g.* the electric or the magnetic fields on flat and curved space-times, or other bilinear combination of fields. In our case the number operator is made out of with fermionic bilinears, which can be associated to physical observable after smearing out coordinate-dependence on space-time regions. Actually, because of the time translation invariance of de-Sitter, smearing is performed only on space-coordinates and with constant test-functions. This smearing procedure must be considered as well as the expectation value procedure when evaluating on coherent states the classical limit of conserved charge, such as the number operator.

The fermionic action has the four-fermion interaction-term induced by solving the torsionful part of the connection in terms of the chiral current:

$$\begin{aligned} \sqrt{-g} \mathcal{L}_\psi &= \\ &= \sqrt{-g} \left( -i\bar{\psi} \not{\nabla} \psi + \frac{3}{M_p^2} (\bar{\psi} \gamma^I \gamma_5 \psi)^2 + q \bar{\psi} \gamma^I e_I^\mu \gamma_5 \psi A_\mu \right), \end{aligned} \quad (\text{A4})$$

in which we recall that  $\nabla_\mu$  is the covariant derivative with respect to the Levi-Civita compatible connection and that  $\not{\nabla} = \nabla_\mu e_I^\mu \gamma^I$ . In what follows, we will use co-moving coordinates and recast the Dirac variation in terms of densitized fermions:

$$\tilde{\psi} = \sqrt[4]{-g} \psi, \quad \bar{\tilde{\psi}} = \sqrt[4]{-g} \bar{\psi}. \quad (\text{A5})$$

The equation of motion then decomposes for right-handed  $\psi_R$  and left-handed  $\psi_L$  components in [43]

$$\begin{aligned} -i\nabla_\mu \tilde{\psi}_R + \frac{\Delta_a}{a^3} \frac{6}{M_p^2} \tilde{\psi}_R \delta_\mu^I \bar{\tilde{\psi}} \gamma_I \gamma_5 \tilde{\psi} + q \tilde{\psi}_R A_\mu &= 0, \\ i\nabla_\mu \tilde{\psi}_L + \frac{\Delta_a}{a^3} \frac{6}{M_p^2} \tilde{\psi}_L \delta_\mu^I \bar{\tilde{\psi}} \gamma_I \gamma_5 \tilde{\psi} + q \tilde{\psi}_L A_\mu &= 0, \end{aligned} \quad (\text{A6})$$

in which  $\Delta_a = [\delta_\mu^i(a-1) + 1]$ . In fact, the initial condition on the fermionic fields originates from the axial current, which is proportional to the square of fermionic fields. We find from the initial conditions of the currents,  $J_0^+ \simeq J_0^- \simeq 10^{-25} M_p^3$ , that the four-fermion term are negligible and easily find the solutions of (A6). We focus only on the right handed components  $\psi_R$ , as the solution for  $\psi_L$  will follow in a similar way. With the choice of comoving coordinates and in the Coulomb gauge, we first write the time component of the differential equations, namely  $\nabla_0 \tilde{\psi}_{R/L} = \partial_0 \tilde{\psi}_{R/L} = 0$ . Thus, the time dependent part of the fermionic fields is easily recovered to be constant in  $\tilde{\psi}$ , which means  $\psi \simeq 1/a^{\frac{3}{2}}$ . Since fermionic fields are scalar functions under diffeomorphism, densitized fields continue to be constant in time even if recast in conformal coordinates. The spatial dependence of spinors, which turns out to contribute only for a multiplicative phase factor, can be found at initial time within the approximation of keeping constant the electromagnetic vector field. For the right-handed component we thus find

$$\tilde{\psi}_R = \begin{pmatrix} \xi^1 \\ \xi^2 \end{pmatrix} = \begin{pmatrix} \xi_0^1 e^{-iA_0^+ x_+} + i \frac{\xi_0^2 H}{4A_0^+} e^{iA_0^+ x_+} \\ \xi_0^2 e^{iA_0^+ x_+} \end{pmatrix}, \quad (\text{A7})$$

in which  $\xi_0^1$  and  $\xi_0^2$  are integration constants, and we have introduced  $x^\pm = \frac{1}{2}(x_1 \pm ix_2)$ . In a similar way, we find

$$\tilde{\psi}_L = \begin{pmatrix} \chi^1 \\ \chi^2 \end{pmatrix} = \begin{pmatrix} \chi_0^1 e^{iA_0^- x_-} \\ \chi_0^2 e^{-iA_0^- x_-} - i \frac{\chi_0^1 H}{4A_0^-} e^{iA_0^- x_-} \end{pmatrix}. \quad (\text{A8})$$

Notice also that the contribution from the fermionic dynamics to the energy-momentum tensor, which in comoving coordinates is proportional to  $\tilde{\psi}^\dagger \gamma_5 \nabla_0 \tilde{\psi}$ , will vanish already at the classical level, consistently with our initial approximation  $\mathcal{F}_5^0 = 0$ .

Dirac fields are fundamental quantum fields and undergo second quantization through the canonical anti-commutation relations

$$\{\bar{\psi}(\vec{x}, t), \psi(\vec{y}, t)\} = i\delta(\vec{x}, \vec{y})$$

that are imposed on space-like surfaces. As a consequence, conservation of the  $\bar{\psi} \gamma^J \psi$  current implies the constancy in time of the number operator. Indeed the time behavior found for the Dirac fields together with the plane-waves expansion of fields (on the space of the classical solution of the Dirac equation) and the covariant integration procedure ensures the constancy in cosmological time of the number operator. In order to derive a semiclassical quantity that could be properly associated with the classical concept of energy-density entering the Einstein equations, an averaging procedure on semiclassical coherent states of the fermionic Fock space has to be considered.

We consider a quantum mechanical system of one-component spinor of different flavors  $a$ , which we still label as  $\psi^a$  and subject to equal-time cononical anti-commutation relations

$$\{\psi^a, \psi_b^\dagger\} = \delta_b^a, \quad \{\psi^a, \psi^b\} = \{\psi_a^\dagger, \psi_b^\dagger\} = 0. \quad (\text{A9})$$

Thus we can construct an algebra of creation  $\psi_a^\dagger$  and annihilation  $\psi_a$  operators acting on  $|0\rangle$ , the vacuum state of the Fock fermionic space that is such that  $\psi^a|0\rangle = 0$  and  $0 = \langle 0|\psi_a^\dagger$ . We further introduce Grassmann variables  $\eta^a$ , which label the eigenvalues of the Ladder operators and whose square absolute value  $\bar{\eta}_a \eta^a$  has the meaning of expectation value of the number operator[44]. As a consequence, we find that fermionic coherent states are defined by

$$|\eta\rangle = e^{\psi_a^\dagger \eta^a} |0\rangle, \quad \langle \bar{\eta}| = \langle 0| e^{\psi_a \bar{\eta}^a}, \quad (\text{A10})$$

and hence fulfill the relations

$$\psi^a |\eta\rangle = \eta^a |\eta\rangle, \quad \langle \bar{\eta}| \bar{\eta}_a = \langle \bar{\eta}| \psi_a^\dagger, \quad (\text{A11})$$

having inner-product  $\langle \bar{\eta}|\eta\rangle = e^{\bar{\eta}_a \eta^a}$ . Thus, without a further normalization, we would find  $\langle \eta^a | \psi^\dagger \psi | \eta\rangle = \bar{\eta}_a \eta^a e^{\bar{\eta}_a \eta^a}$ .

This construction can be applied to the second quantization of the Dirac fields  $\psi(\vec{x}, t)$  and  $\bar{\psi}(\vec{x}, t)$ . Let us consider the massless Dirac field, but for facility of treatment let us write the Dirac equation in conformal coordinates [24]:

$$(i\partial - e\mathcal{A})(a^2(\eta)\psi(\vec{x}, \eta)) = 0, \quad (\text{A12})$$

implying the expansion in terms of conserved space-momenta  $\vec{p}$  and related frequency  $\omega_{\vec{p}} = |\vec{p}|$

$$\begin{aligned} a^2(\eta)\psi(\vec{x}, \eta) &= \sum_{r\vec{p}} \frac{1}{\sqrt{2V\omega_{\vec{p}}}} [c_r(\vec{p}) u_r(\vec{p}) e^{-ip \cdot x} + d_r^\dagger(\vec{p}) v_r(\vec{p}) e^{ip \cdot x}], \\ a^2(\eta)\bar{\psi}(\vec{x}, \eta) &= \sum_{r\vec{p}} \frac{1}{\sqrt{2V\omega_{\vec{p}}}} [d(\vec{p}) \bar{v}_r(\vec{p}) e^{-ip \cdot x} + c_r^\dagger(\vec{p}) \bar{u}_r(\vec{p}) e^{ip \cdot x}] = a^2(\eta)\psi^\dagger(\vec{x}, \eta)\gamma_0, \end{aligned} \quad (\text{A13})$$

in which  $V$  is a coordinate-space normalization volume, the notation  $\bar{u}_r(\vec{p}) = u_r^\dagger(\vec{p}) \gamma_0$  and  $\bar{v}_r(\vec{p}) = v_r^\dagger(\vec{p}) \gamma_0$  has been meant for the spinor  $u_r(\vec{p})$  and  $v_r(\vec{p})$ , and finally the following orthonormality relations have been chosen

$$\begin{aligned} u_e^\dagger(\vec{p}) u_s(\vec{p}) &= v_e^\dagger(\vec{p}) v_s(\vec{p}) = \omega_{\vec{p}} \delta_{rs}, \\ u_e^\dagger(\vec{p}) v_s(-\vec{p}) &= 0. \end{aligned} \quad (\text{A14})$$

We can now proceed and quantize as in (A11) the Fourier coefficient  $a_{\vec{p}}$  and  $a_{\vec{p}}^\dagger$  entering the expansion of fields, and hence define for each momentum  $\vec{p}$  of the Fourier expansion a copy of the fermionic Fock space, with a vacuum state such that  $a_{\vec{p}}|0\rangle = 0$  for all  $\vec{p}$ . Coherent states are therefore defined as in (A10), namely

$$|\xi_{\vec{p}}\rangle = e^{a_{\vec{p}}^\dagger \xi_{\vec{p}}} |0\rangle, \quad \langle \bar{\xi}_{\vec{p}}| = \langle 0| e^{a_{\vec{p}} \bar{\xi}_{\vec{p}}}, \quad (\text{A15})$$

in which we have now hidden the flavor index  $a$ . We can also construct normalized coherent states  $|\Xi\rangle$  considering the product  $|\Xi\rangle = \prod_{\vec{p}} |\xi_{\vec{p}}\rangle$  over each mode entering



the Fourier expansion of the Dirac fields, and then take the expectation value of the number operator over such states. Thus the expectation value of the fermion number density operator evaluated within the coherent state is given by

$$\begin{aligned} n(t) &= \frac{N}{\mathcal{V}} = \langle \Xi | \frac{\hat{N}}{\mathcal{V}} | \Xi \rangle = \\ &= \frac{\langle \Xi | \int d^3x a^3(t) \bar{\Psi}(\vec{x}, t) \gamma_0 \Psi(\vec{x}, t) | \Xi \rangle}{\langle \Xi | \int d^3x a^3(t) | \Xi \rangle} = \frac{n_0}{a^3(t)}, \end{aligned} \quad (\text{A16})$$

in which we have introduced the total difference number of fermions  $N$ , its density number  $n$ , and the time-independent part of the density number  $n_0$ .

Notice that from (A13) it follows that Ladder operators too must be densitized. Indeed  $c_r(\vec{p}), d_r(\vec{p}) \rightarrow c_r(\vec{p})a^{-2}, d_r(\vec{p})a^{-2}$  and the same happens to their Hermitian conjugated operators, *i.e.*  $c_r^\dagger(\vec{p}), d_r^\dagger(\vec{p}) \rightarrow c_r^\dagger(\vec{p})a^{-2}, d_r^\dagger(\vec{p})a^{-2}$ . As a consequence, eigenvalues of inflationary coherent states are now shifted to  $\xi(t)\vec{p} = \xi_{\vec{p}} a^2(t)$ . We emphasize that inflationary coherent states have been defined in [30, 31] as excited states of the Bunch-Davies vacuum  $|0\rangle_{BD}$ . Thus, coherent state defined by  $|\Xi_a\rangle = \prod_{\vec{p}} |\xi_{\vec{p}} a^2\rangle$  ensures that the expectation value of the bilinear current of the Dirac fields scales as a constant. Recalling indeed that  $\mathcal{J}_\mu = \Omega(\bar{\psi}\gamma^5\gamma^\mu e^\mu_\nu\psi)$ ,  $\Omega$  summarizing the expectation value procedure on the state  $|\Xi_a\rangle$ , and that in conformal coordinates the inverse vierbein is  $e^\mu_I = a^{-1}\delta^\mu_I$ , we find that  $\mathcal{J}_i = a^{-1}\Omega(\bar{\psi}_0\gamma_5\gamma_i\psi_0) = J_i/a$ , with  $J_i$  constant in time.

## Appendix B: Stability Analysis of Vector Perturbations

In this model a constant Horizon size mode of the vector fields participate initiating inflation. However, it is important to check whether during inflation spatial perturbations in the vector fields,  $\delta A_i$ , lead to instabilities. Perturbations to the spatially flat FLRW metric in conformal coordinates are written as

$$\begin{aligned} ds^2 &= a^2(\eta) \{ -(1+2\Phi)d\eta^2 - 2B_i\delta dx^i + \\ &+ [(1-2\Psi)\delta_{ij} + 2h_{ij}]dx^i dx^j \}, \end{aligned} \quad (\text{B1})$$

with  $\Phi$  and  $\Psi$  scalar,  $B_i$  vector and  $h_{ij}$  tensor Bardeen potentials accounting for perturbations to the metric tensor. It is well known that vector fields will source anisotropic stress in the energy momentum tensor and influence the gauge invariant vector mode Bardeen variable,  $B_i$ . In this appendix we derive the equations and show that eventhough the vector modes in  $A_i$  are generated during inflation, there are not instabilities because the solutions to perturbed Einstein equations yield a red-shifting vector mode in our model,  $B \sim \frac{1}{a^2}$ .

We start from

$$\begin{aligned} T_j^i &= \sum_{K=1}^N \left[ -\delta_j^i A_\rho^K \mathcal{J}_5^\rho + F_{\rho\alpha}^K F_{\sigma(j}^K g^{i)\rho} g^{\alpha\sigma} - \frac{1}{4} \delta_j^i g^{\alpha\rho} g^{\beta\sigma} F_{\alpha\beta}^K F_{\rho\sigma}^K + \right. \\ &\quad \left. - A_{(j}^K \mathcal{J}_5^{i)} \right] = \sum_{K=1}^N \left[ -\delta_j^i A_\rho^K \mathcal{J}_5^\rho + \partial_{[\rho} A_{\alpha]}^K \partial_{[\sigma} A_{(j)}^K g^{i)\rho} g^{\alpha\sigma} + \right. \\ &\quad \left. - \frac{1}{4} \delta_j^i g^{\alpha\rho} g^{\beta\sigma} \partial_{[\alpha} A_{\beta]}^K \partial_{[\rho} A_{\sigma]}^K - A_{(j}^K \mathcal{J}_5^{i)} \right] \end{aligned} \quad (\text{B2})$$

and varying in  $A^K$  we obtain

$$\begin{aligned} \delta T_j^i &= \sum_{K=1}^N \left[ + \left( \partial_{[\rho} \delta A_{\alpha]}^K \partial_{[\sigma} A_{(j)}^K + \partial_{[\rho} A_{\alpha]}^K \partial_{[\sigma} \delta A_{(j)}^K \right) g^{i)\rho} g^{\alpha\sigma} \right. \\ &\quad \left. - \frac{1}{4} \delta_j^i g^{\alpha\rho} g^{\beta\sigma} \left( \partial_{[\alpha} \delta A_{\beta]}^K \partial_{[\rho} A_{\sigma]}^K + \partial_{[\alpha} A_{\beta]}^K \partial_{[\rho} \delta A_{\sigma]}^K \right) + \right. \\ &\quad \left. - \delta A_{(j}^K \mathcal{J}_5^{i)} - \delta_j^i \delta A_\rho^K \mathcal{J}_5^\rho \right]. \end{aligned} \quad (\text{B3})$$

We can further rewrite  $\delta T_j^i$  expressing  $\delta A_i^K = \bar{\delta} A_i^K + \partial_i \chi^K$  (we are here following [8])

$$\begin{aligned} \delta T_j^i &= \sum_{K=1}^N \left\{ -\delta_j^i [\delta A_0^K \mathcal{J}_5^0 + (\bar{\delta} A_l^K + \partial_l \chi^K) \mathcal{J}_5^l] - \delta A_{(j}^K \mathcal{J}_5^{i)} + \right. \\ &\quad \left. + \frac{1}{a^2} \left( \partial_{[s} \delta A_{\alpha]}^K \partial_{[\sigma} A_{(j)}^K \delta^{i)s} + \partial_{[s} A_{\alpha]}^K \partial_{[\sigma} \delta A_{(j)}^K \delta^{i)s} \right) g^{\alpha\sigma} + \right. \\ &\quad \left. - \frac{1}{4} \delta_j^i \frac{1}{a^4} \left( \partial^{[\alpha} \delta A_{\beta]}^K \partial_{[\alpha} A^{K\beta]} + \partial^{[\alpha} A_{\beta]}^K \partial_{[\alpha} \delta A^{K\beta]} \right) \right\} = \\ &= \sum_{K=1}^N \left\{ -\delta_j^i [\delta A_0^K \mathcal{J}_5^0 + (\bar{\delta} A_l^K + \partial_l \chi^K) \mathcal{J}_5^l] + \right. \\ &\quad \left. + \frac{1}{a^4} \left( \partial^{[\alpha} A_{(j)}^K \partial_{[i]} \delta A_{\alpha]}^K + \partial^{[\alpha} \delta A_{(j)}^K \partial_{[i]} A_{\alpha]}^K \right) - \delta A_{(j}^K \mathcal{J}_5^{i)} + \right. \\ &\quad \left. - \frac{1}{4} \delta_j^i \frac{1}{a^4} \left( \partial^{[\alpha} \delta A_{\beta]}^K \partial_{[\alpha} A^{K\beta]} + \partial^{[\alpha} A_{\beta]}^K \partial_{[\alpha} \delta A^{K\beta]} \right) \right\}. \end{aligned} \quad (\text{B4})$$

Again, using  $\delta A_i^K = \bar{\delta} A_i^K + \partial_i \chi^K$ , we find

$$\begin{aligned} \delta T_j^i &= \sum_{K=1}^N \left\{ -\delta_j^i [\delta A_0^K \mathcal{J}_5^0 + (\bar{\delta} A_l^K + \partial_l \chi^K) \mathcal{J}_5^l] + \right. \\ &\quad \left. + \frac{1}{a^4} \left[ \left( \partial_0 A_{(j}^K - \partial_{(j} A_0^K \right) \left( \partial_{i)} \delta A_0^K - \partial_0 \delta A_{i)}^K \right) + \right. \right. \\ &\quad \left. \left. + \left( \partial_s A_{(j}^K - \partial_{(j} A_s^K \right) \partial_{[i]} \left( \bar{\delta} A_{s]}^K + \partial_{s]} \chi^K \right) + \right. \right. \\ &\quad \left. \left. + \left( \partial_0 \left( \partial_{(j} \chi^K + \bar{\delta} A_{(j)}^K \right) - \partial_{(j} \delta A_0^K \right) \left( \partial_{i)} A_0^K - \partial_0 A_{i)}^K \right) + \right. \right. \\ &\quad \left. \left. + \partial_{[s} \left( \bar{\delta} A_{(j)}^K + \partial_{(j)} \chi^K \right) \partial_{[i]} A_{s]}^K \right] - \left( \partial_{(j} \chi^K + \bar{\delta} A_{(j)}^K \right) \mathcal{J}_5^{i)} + \right. \\ &\quad \left. - \frac{1}{2a^4} \delta_j^i \left[ 2 \left( \partial_0 \left( \bar{\delta} A_s^K + \partial_s \chi^K \right) - \partial_s \delta A_0^K \right) \partial_{[0} A_{s]}^K + \right. \right. \\ &\quad \left. \left. + \partial_{[s} \left( \partial_{l]} \chi^K + \bar{\delta} A_{l]}^K \right) \partial_{[s} A_{l]}^K \right] \right\}. \end{aligned} \quad (\text{B5})$$

We now assume  $\mathcal{J}_5^0 = 0$  and  $\mathcal{J}_5^i = \frac{1}{a} J^i$ , with  $J^i$  a con-

stant three-vector, and rewrite  $\delta T_j^i$  as

$$\begin{aligned} \delta T_j^i = & \sum_{K=1}^N \left\{ -\delta_j^i \frac{1}{a} (\bar{\delta} A_l^K + \partial_l \chi^K) J^l - \frac{1}{a} (\partial_{(j} \chi^K + \bar{\delta} A_{(j}^K) J^{i)} + \right. \\ & + \frac{1}{a^4} \left[ (\partial_0 A_{(j}^K - \partial_{(j} A_0^K) (\partial_{i)} \delta A_0^K - \partial_0 \delta A_{i)}^K) + \right. \\ & + (\partial_s A_{(j}^K - \partial_{(j} A_s^K) \partial_{[i]} (\bar{\delta} A_{s]}^K + \partial_{s]} \chi^K) + \\ & + (\partial_0 (\partial_{(j} \chi^K + \bar{\delta} A_{(j}^K) - \partial_{(j} \delta A_0^K) (\partial_{i)} A_0^K - \partial_0 A_{i)}^K) + \\ & + \partial_{[s} (\bar{\delta} A_{(j]}^K + \partial_{(j]} \chi^K) \partial_{[i]} A_{s]}^K] + \\ & - \frac{1}{2a^4} \delta_j^i \left[ 2 (\partial_0 (\bar{\delta} A_s^K + \partial_s \chi^K) - \partial_s \delta A_0^K) \partial_{[0} A_{s]}^K + \right. \\ & \left. \left. + \partial_{[s} (\partial_{l]} \chi^K + \bar{\delta} A_{l]}^K) \partial_{[s} A_{l]}^K \right] \right\}. \end{aligned} \quad (B6)$$

We now fix the gauge and impose  $A_0^K = 0$ , finding

$$\begin{aligned} \delta T_j^i = & \sum_{K=1}^N \left\{ -\delta_j^i \frac{1}{a} (\bar{\delta} A_l^K + \partial_l \chi^K) J^l - \frac{1}{a} (\partial_{(j} \chi^K + \bar{\delta} A_{(j}^K) J^{i)} + \right. \\ & + \frac{1}{a^4} \left[ \partial_0 A_{(j}^K (\partial_{i)} \delta A_0^K - \partial_0 \delta A_{i)}^K) + \right. \\ & + (\partial_s A_{(j}^K - \partial_{(j} A_s^K) \partial_{[i]} (\bar{\delta} A_{s]}^K + \partial_{s]} \chi^K) + \\ & - (\partial_0 (\partial_{(j} \chi^K + \bar{\delta} A_{(j}^K) - \partial_{(j} \delta A_0^K) \partial_0 A_{i)}^K + \\ & + \partial_{[s} (\bar{\delta} A_{(j]}^K + \partial_{(j]} \chi^K) \partial_{[i]} A_{s]}^K] + \\ & - \frac{1}{2a^4} \delta_j^i \left[ 2 (\partial_0 (\bar{\delta} A_s^K + \partial_s \chi^K) - \partial_s \delta A_0^K) \partial_0 A_s^K + \right. \\ & \left. \left. + \partial_{[s} (\partial_{l]} \chi^K + \bar{\delta} A_{l]}^K) \partial_{[s} A_{l]}^K \right] \right\}. \end{aligned} \quad (B7)$$

Following again [8], we write

$$\delta T_j^i = \delta p_A \delta_j^i + p_A \Pi_j^i \quad (B8)$$

in which

$$\Pi_j^i = {}^{(s)}\Pi_j^i + {}^{(v)}\Pi_j^i + {}^{(t)}\Pi_j^i \quad (B9)$$

and

$${}^{(s)}\Pi_j^i = \left( \partial^i \partial_j - \frac{1}{3} \delta_j^i \partial^l \partial_l \right) \Pi \quad (B10)$$

with  $\Pi$  scalar,

$${}^{(v)}\Pi_j^i = \partial_j \Pi^i \quad (B11)$$

with  $\Pi^i$  transverse vector,

$${}^{(t)}\Pi_j^i \quad (B12)$$

traceless and tranverse tensor.

Thus

$$\begin{aligned} \delta p_A = & \sum_{K=1}^N \left\{ -\frac{1}{a} (\bar{\delta} A_i^K + \partial_i \chi^K) J^i + \right. \\ & - \frac{1}{2a^4} \left[ 2 (\partial_0 (\bar{\delta} A_s^K + \partial_s \chi^K) - \partial_s \delta A_0^K) \partial_0 A_s^K + \right. \\ & \left. \left. + \partial_{[s} (\partial_{l]} \chi^K + \bar{\delta} A_{l]}^K) \partial_{[s} A_{l]}^K \right] \right\} + \frac{1}{3} (p_A \Pi_j^i) \delta_j^i \end{aligned} \quad (B13)$$

with

$$\begin{aligned} p_A \Pi_j^i = & \sum_{K=1}^N \left\{ -\frac{1}{a} (\partial_{(j} \chi^K + \bar{\delta} A_{(j}^K) J^{i)} + \right. \\ & + \frac{1}{a^4} \left[ \partial_0 A_{(j}^K (\partial_{i)} \delta A_0^K - \partial_0 \delta A_{i)}^K) + \right. \\ & + (\partial_s A_{(j}^K - \partial_{(j} A_s^K) \partial_{[i]} (\bar{\delta} A_{s]}^K + \partial_{s]} \chi^K) + \\ & - (\partial_0 (\partial_{(j} \chi^K + \bar{\delta} A_{(j}^K) - \partial_{(j} \delta A_0^K) \partial_0 A_{i)}^K + \\ & \left. \left. + \partial_{[s} (\bar{\delta} A_{(j]}^K + \partial_{(j]} \chi^K) \partial_{[i]} A_{s]}^K \right] \right\}, \end{aligned} \quad (B14)$$

and therefore

$$\begin{aligned} p_A \Pi_j^i = & \sum_{K=1}^N \left\{ -\frac{2}{a} (\partial_i \chi^K + \bar{\delta} A_i^K) J^i + \right. \\ & + \frac{2}{a^4} \left[ \partial_0 A_i^K (\partial_i \delta A_0^K - \partial_0 \delta A_i^K) + \right. \\ & + (\partial_s A_i^K - \partial_i A_s^K) \partial_{[i} (\bar{\delta} A_{s]}^K + \partial_{s]} \chi^K) + \\ & - (\partial_0 (\partial_i \chi^K + \bar{\delta} A_i^K) - \partial_i \delta A_0^K) \partial_0 A_i^K + \\ & \left. \left. + \partial_{[s} (\bar{\delta} A_{i]}^K + \partial_{i]} \chi^K) \partial_{[i} A_{s]}^K \right] \right\} \\ = & \sum_{K=1}^N \left\{ -\frac{2}{a} (\partial_i \chi^K + \bar{\delta} A_i^K) J^i + \right. \\ & + \frac{4}{a^4} \left[ \partial_0 A_i^K (\partial_i \delta A_0^K - \partial_i \dot{\chi}^K - \bar{\delta} \dot{A}_i^K) + \right. \\ & \left. \left. + \partial_{[s} \bar{\delta} A_{i]}^K \partial_{[i} A_{s]}^K \right] \right\}. \end{aligned} \quad (B15)$$

**${}^{(v)}\Pi_j^i$  components of the perturbed energy-momentum tensor**

We want here to obtain the  ${}^{(v)}\Pi_j^i$  components of the perturbed energy-momentum tensor  $\Pi_j^i$  and prove that  ${}^{(v)}\Pi_j^i = 0$ . We consider then

$$\begin{aligned} p_A \Pi_j^i = & \sum_{K=1}^N \left\{ -\frac{2}{a} (\partial_i \chi^K + \bar{\delta} A_i^K) J^i + \right. \\ & \left. + \frac{4}{a^4} \left[ \partial_0 A_i^K (\partial_i \delta A_0^K - \partial_i \dot{\chi}^K - \bar{\delta} \dot{A}_i^K) + \partial_{[s} \bar{\delta} A_{i]}^K \partial_{[i} A_{s]}^K \right] \right\} \end{aligned}$$

and focus on its components involving  $\partial_i$  derivatives. For simplicity, we work at  $K$  fixed

$$\begin{aligned} \frac{4}{a^4} \left[ \partial_0 A_i^K (\partial_i \delta A_0^K - \partial_i \dot{\chi}^K - \bar{\delta} \dot{A}_i^K) + \partial_{[s} \bar{\delta} A_{i]}^K \partial_{[i} A_{s]}^K + \right. \\ \left. - \frac{2}{a} (\partial_i \chi^K + \bar{\delta} A_i^K) J^i \right] \end{aligned} \quad (B16)$$

and analyze each term involved in this latter equation.

- About the first term in (B16) we notice that, because of the constancy of  $J^i$ , we can rewrite it as

$\partial_i (\chi^K J^i)$ . But if this term belong to  $^{(v)}\Pi_j^i$ , then we should have

$$\partial_i (\chi^K J^i) = 0, \quad (\text{B17})$$

which happens iff  $\partial_i \chi^K = 0$ , *i.e.* if the term is not present at all and  $\delta A_i^K$  does not include perturbations coming from the gradient of a set of scalar functions  $\chi^K$ . We can conclude that this term can not contribute to  $^{(v)}\Pi_j^i$ .

- We apply the same reasoning to the third and the forth term, assuming the transverse Coulomb gauge for  $A^K$ , namely  $\partial_i A_i^K = 0$ . Then

$$\begin{aligned} \dot{A}_i^K (\delta A_0^K - \dot{\chi}^K) &= \\ &= \partial_i \left[ \dot{A}_i^K (\delta A_0^K - \dot{\chi}^K) \right] - (\partial_i \dot{A}_i^K) (\delta A_0^K - \dot{\chi}^K) = \\ &= \partial_i \left[ \dot{A}_i^K (\delta A_0^K - \dot{\chi}^K) \right]. \end{aligned} \quad (\text{B18})$$

If this term is the contraction of the  $i$  and  $j$  indices of term belonging to  $^{(v)}\Pi_j^i$ , then it has to be zero by definition of  $^{(v)}\Pi_j^i$ , *i.e.*

$$\partial_i \left[ \dot{A}_i^K (\delta A_0^K - \dot{\chi}^K) \right] = 0 \quad \text{iff} \quad \delta A_0^K = \dot{\chi}^K. \quad (\text{B19})$$

But this condition would imply that the term which should contribute to  $^{(v)}\Pi_j^i$  is identically zero.

- The remaining term involving derivative in  $\partial_i$  that we should consider is the last one of (B16). We can rewrite this last term, again using  $\partial_i A_i^K = 0$ , as

$$\begin{aligned} \partial_{[s} \bar{\delta} A_{i]}^K \partial_{[i} A_{s]}^K &= \\ &= \partial_i \left[ \left( \partial_{[s} A_{i]}^K \right) \bar{\delta} A_s^K \right] - \partial_i \left( \partial_{[s} A_{i]}^K \right) \bar{\delta} A_s^K = \\ &= \partial_i \left[ \left( \partial_{[s} A_{i]}^K \right) \bar{\delta} A_s^K \right] - \partial_i (\partial_s A_i^K - \partial_i A_s^K) \bar{\delta} A_s^K. \end{aligned} \quad (\text{B20})$$

We focus now on the last term in (B20). Using again  $\partial_i A_i^K = 0$ , we find

$$\begin{aligned} -\partial_i (\partial_s A_i^K - \partial_i A_s^K) &= -\partial_s \partial_i A_i^K + \partial_i \partial_i A_s^K = \\ &= \partial_i \partial_i A_s^K \neq 0, \end{aligned} \quad (\text{B21})$$

where we have used in the last hand side the equation of motion for  $A^K$ . Thus this last contribution

in (B20) can not result to be the trace in  $i$  and  $j$  of a term belonging to  $^{(v)}\Pi_j^i$ .

The analysis of the points above ensure us that  $^{(v)}\Pi_j^i = 0$ . As a consequence, within the same notation of [8], we find for the vector perturbations  $B^i$  that

$$\partial_j \dot{B}^i + 2 \frac{\dot{a}}{a} \partial_j B^i = 8\pi G a^2 p_A^{(v)} \Pi_j^i = 0, \quad (\text{B22})$$

implying

$$\partial_j B^i \sim \frac{1}{a^2}. \quad (\text{B23})$$

The differential equation regulating the evolution of the transverse vector perturbations  $\delta A^i$  reads

$$\partial_0^2 \delta A_i - \partial_j \partial^j \delta A_i - \partial_0 A^j \partial_j B_i - 2\partial_0 h_{ij} \partial_0 A^j - \delta_i = 0 \quad (\text{B24})$$

in which  $\delta_i$  is defined by means of the decomposition of

$$2\Phi \partial_0^2 A_i + \partial_0 (\Phi + \Psi) \partial_0 A_i = \partial_i \delta + \delta_i \quad (\text{B25})$$

in a scalar  $\delta$  and  $\delta_i$ , which is a transverse vector. Notice that, assuming  $\Phi = -\Psi$ , (B25) implies  $\delta \simeq e^{i\vec{k} \cdot \vec{x}} f(\eta)$ , with  $f(\eta)$  generic. When  $\Phi = 0$ , then both  $\delta_i$  and  $\delta$  are vanishing, and the only vector perturbations  $B_i$  and tensor perturbations  $h_{ij}$  enter (B24). These latter are determined by

$$\partial_0^2 h_{ij} + 2 \frac{\partial_0 a}{a} \partial_0 h_{ij} - \partial^k \partial_k h_{ij} = 8\pi G p_A^{(t)} \Pi_{ij} \simeq 0, \quad (\text{B26})$$

in which we are disregarding the anisotropic stress  $^{(t)}\Pi_{ij}$ . The solution to (B26) is given by  $h_{ij} \simeq \frac{\text{BesselJ}(1, i/a(\eta))}{a(\eta)}$ . Thus, the evolution of (B24) is sourced by terms that are suppressed in  $a(\eta)$ , and the stability of the solution for  $A_i(\eta)$ , and therefore  $a(\eta)$ , is recovered.

## Acknowledgements

We would like to thank Robert Brandenberger, Robert Caldwell, Larry Ford, S. James Gates Jr., Alan Guth, Elias Kiritsis, Justin Khoury, Matthew Kleban, Paul Langacker, Alessio Notari, Mikhail Shaposhnikov, Lorenzo Sorbo, Herman Verlinde and Edward Witten for enlightening discussions.

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- [32] We thank Jim Gates for pointing out this fact to us.
- [33] It is possible that this scalar field is not fundamental and is instead analogous to a gauge singlet quark condensate such as the  $\eta$  meson. It is well known that the  $\eta$  particle couples to the Abelian Chern-Simons term to address the  $U(1)$  problem.
- [34] We will be justified to not include the coupling of the superhorizon to the vector modes  $V_i$ , since we will show that spatial anisotropies on superhorizon scales will be suppressed by  $\sqrt{N}$ . We thank Justin Khoury for raising this point.
- [35] Even if  $\beta_k$  ( $k > \dot{\theta}/M_*$ ) is imaginary and the exponential of the general solution in (9) and  $\Psi$  become oscillatory, the gauge field still grows because of the  $\tilde{J}_-/[2H^2(1-H\eta)]$  term in  $\Xi[H, \beta, \tilde{J}_-, \eta]$ , due to the back-reaction provided by the gravitational field.
- [36] It has been shown by Ford [2] that vector fields necessarily generate an anisotropic metric arising from off-diagonal components of the energy momentum tensor. The anisotropic parts of the energy-momentum tensor due to the field-strength are  $T_{0i} = S_i$ , where  $\vec{S} = \vec{E} \times \vec{B}$  is the Poynting vector, and  $T_{ij} = E_i E_j + B_i B_j - (\vec{E}^2 + \vec{B}^2)\delta_{ij}/2$ . Likewise, we will find that values of the gauge field necessary to generate inflation in the isotropic part of the energy-momentum tensor yield electric and magnetic components that are perturbations. This occurs because the derivatives that act on  $A$  to generate the electric and magnetic fields are suppressed by  $a^{-4}$ . Due to the growth of the gauge field the electric field will redshift as  $a^{-2}$ . We thank Robert Brandenberger for bringing this to our attention.
- [37] Numerical simulations show that  $A \cdot \mathcal{J}$  is monotonically decreasing in time during inflation and has a maximum relative variation of  $\simeq 0.5$ .
- [38] We choose the initial conditions  $\tilde{A}_-^0 = \tilde{A}_+^0 = 0$ .
- [39] For more details on the role of the Chern-Simons term in cosmology we refer the reader to [19–21].
- [40] We thank the referee for raising this crucial possibility.
- [41] We are grateful to the referee for raising the point of non-standard charge assignments in our model.
- [42] Since we have extended the gauge sector to include  $N$  copies of gauge field, we are assuming that these abelian factors only carry solely leptonic charge in the minimal standard model. In general we have the freedom to couple the gauge fields to other states that are not charged under the weak-hypercharge assignment (such as a dark sector), but this falls outside the scope of this present work. We thank the referee for raising this crucial point, since non-standard assignments may provide a mechanism of dark-matter production at the end of inflation.
- [43] In general, fermionic coupling to general relativity induces torsion. Inclusion of a four-fermion term will eliminate the torsion and yield metric compatibility. However, the contribution to the four-fermion interaction is negligible, as it is proportional to  $\mathcal{J}_5^\mu$ , whose components are

suppressed by a factor of  $a$ . Furthermore, the introduction of densitized Dirac fields will introduce an additional redshift factor proportional to  $\sqrt{-g}$ .

[44] Notice that, being  $\eta^a$  a Grassmann variable, we have that the vacuum states commutes with Grassmann variables,

namely  $|0\rangle\eta^a = \eta^a|0\rangle$  and  $|0\rangle\bar{\eta}^a = \bar{\eta}^a|0\rangle$ . The same property holds for  $\langle 0|$ . Moreover, we can say that  $\bar{\eta}^a$  is the complex conjugate Grassmann variable associated to the Grassmann variable  $\eta^a$ .